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Materials Selection to Resist Creep [and Discussion]

M. F. Ashby, C. A. Abel and M. J. Goulette

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Materials selection to resist creep

By M. F. Ashby and C. A. Abel

Engineering Design Centre, Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, UK

A design-led procedure is developed for the selection of materials to resist creep, creep fracture, creep relaxation and creep buckling. It is an extension of a successful procedure for room temperature design which uses performance indices and materials-selection charts. The extension requires the definition of a 'design strength', σ_D, which characterizes material response under conditions imposed by the design. Materials are ranked by an optimization procedure which combines σ_D with other properties (such as density, or cost, or stiffness) to isolate the subset which best meet the design specification.

1. Introduction

How are materials chosen to avoid failure by creep at high temperatures (figure 1)? Very largely by experience. Polymers can be used at room temperature, but, with only a few exceptions, not above 100 °C. The most creep resistant of aluminium alloys are good to about 200 °C; titanium alloys to 600 °C; stainless steels to 850 °C, and so on. But optimal selection requires much more than this. The choice depends not only on material properties, but on the mode of loading (tension, bending, torsion, internal pressure), on the failure criterion (excessive deflection, fracture, relaxation of stress, bucking, etc.) and on the optimization objective (minimizing weight, or cost, or maximizing life). A designer, not himself a specialist on creep, has no easy way to identify the subset of materials best suited to his needs, or to predict the ways in which a change in the design might influence the choice. In short, we lack a systematic procedure for selecting materials for use at high temperatures.

There *is* a systematic method for selecting materials for mechanical design at room temperature. Could it be extended to high? To answer this, we first examine how the method works.

2. Material selection for room temperature design

The method uses two key ideas: *performance indices* and *material-property space*.

A performance index is a group of material properties which characterizes some aspect of the performance of a component (Crane & Charles 1984; Ashby 1992). Selecting a material with the largest value of the appropriate index maximizes this aspect of performance. The indices are derived from models of the function of the component. The best materials for making a light strong *tie* (a tensile

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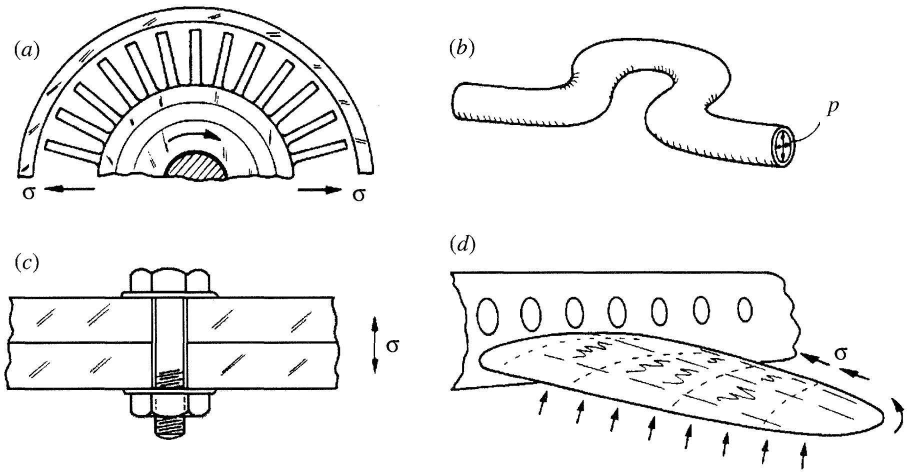

Figure 1. Creep is important in four classes of design: (*a*) displacement-limited, (*b*) failure-limited, (*c*) relaxation-limited and (*d*) buckling-limited.

member) are those which have the highest value of the specific strength:

$$M_1 = \sigma_y/\rho, \tag{2.1}$$

where σ_y is the yield strength and ρ is the density. This will appear obvious; it could have been guessed. But it is dangerous to guess indices. The best material for a light strong *beam*, i.e. a component loaded in bending rather than tension, is that with the greatest value of

$$M_2 = \sigma_y^{2/3}/\rho, \tag{2.2}$$

or, if the width of the beam is fixed but its height can be adjusted (when it is called a *panel*), that with the greatest value of

$$M_3 = \sigma_y^{1/2}/\rho. \tag{2.3}$$

Material indices such as these are derived from models which describe *function*, *objective* and *constraints*. Equation (2.2), for example, is derived as follows. A material is required for a light, strong beam. The beam has a specified length L and a rectangular cross section $b \times h$ such that $b = \alpha h$ (α also specified, so that the shape remains constant) as shown in figure 2*a*. Its function is obvious: it is that of supporting a bending moment. The objective is to minimize its mass m, given by

$$m = AL\rho, \tag{2.4}$$

where $A = bh = \alpha h^2$ is the area of the cross section and ρ is the density of the material of which the beam is made. There are constraints: first, the length L and the proportions α are specified, and second, the beam must not collapse plastically under the load F. Plasticity starts when the stress in the surface of the beam first reaches the yield strength σ_y of the material; plastic collapse occurs when this plasticity penetrates through the entire section to give a plastic hinge. The load F which will just cause collapse is

$$F = C_1 \sigma_y b h^2/L, \tag{2.5}$$

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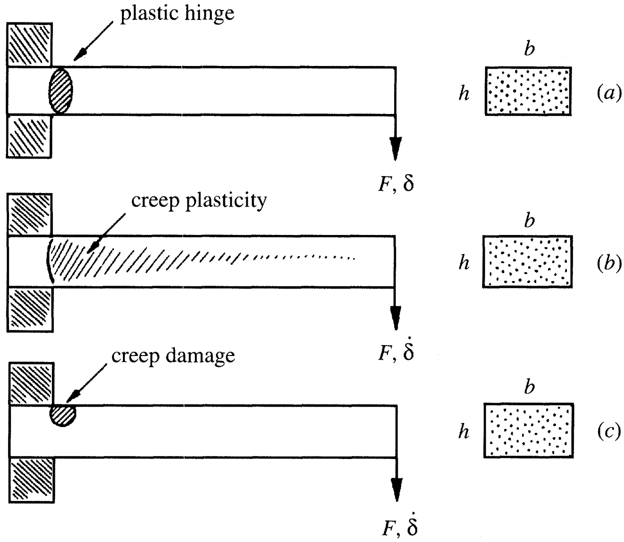

Figure 2. A cantilever beam loaded with an end load F. (*a*) At room temperature a plastic hinge forms where the bending moment is highest. (*b*) At high temperature creep plasticity is distributed. (*c*) Creep fracture starts where the local tensile stress is highest.

where C_1 is a constant which depends only on the load distribution; for the cantilever of figure 2, $C_1 = \frac{1}{4}$. Other supports or other distributions of load change C_1 but nothing else. The beam can be made lighter by reducing the area A of the cross section, but if it is reduced too much the beam will collapse under the load F; we are free to choose A so as to reduce the mass, provided the constraint is still met. Substituting for A in (2.4) from (2.5) (and using $bh^2 = A^{3/2}/\alpha^{1/2}$ and $C_1 = \frac{1}{4}$) gives

$$m = \rho L \alpha^{1/3} \{4FL/\sigma_{\mathrm{y}}\}^{2/3}. \tag{2.6}$$

In this equation the quantities L, α and F are fixed by the design. The mass m is minimized by minimizing the remaining term $\rho/\sigma_{\mathrm{y}}^{2/3}$. It is convenient to restate this as a quantity to be maximized: the mass of the beam can be minimized (and performance maximized) by seeking the material with the largest value of the performance index of (2.2). Panels (plates of fixed width loaded in bending) gibe, by a similar route, (2.3).

The differences in the exponents that appear in the three index equations (1, 2/3, and 1/2) drastically change the choice of materials. This is where the idea of material-property space enters: a multi-dimensional space with values of material properties as axes. Sections though this space can be charted ('material-selection charts') and, on to these, the material indices can be plotted. In the example shown as figure 3, the axes are $\log(\sigma_{\mathrm{y}})$ and $\log(\rho)$. When data for a given material class – metals, for instance – are plotted on these axes, it is found that they occupy a field which can be enclosed in a 'balloon'. Ceramics as a class occupy a characteristic field, and so do polymers, elastomers and composites. The fields may overlap but are nonetheless distinct. Individual materials or subclasses (tungsten in the class of metals, or polypropylenes in the class of polymers) appear as little bubbles within each class-balloon, with dimensions defined by the ranges of their properties.

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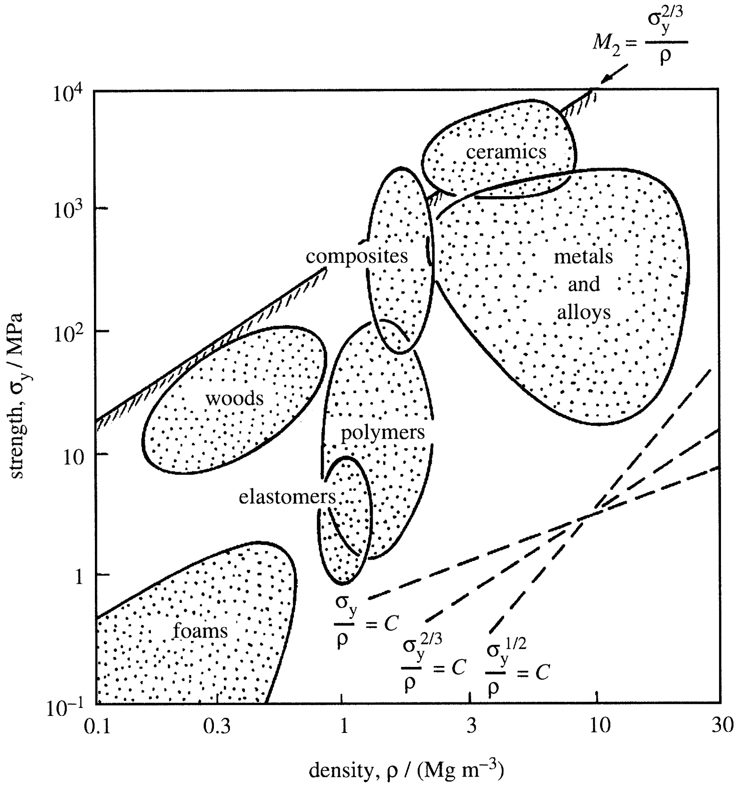

Figure 3. A selection chart for strength-limited design showing the yield strength σ_y plotted against the density ρ on logarithmic scales. A compilation of such charts for room temperature design is given by Ashby (1992).

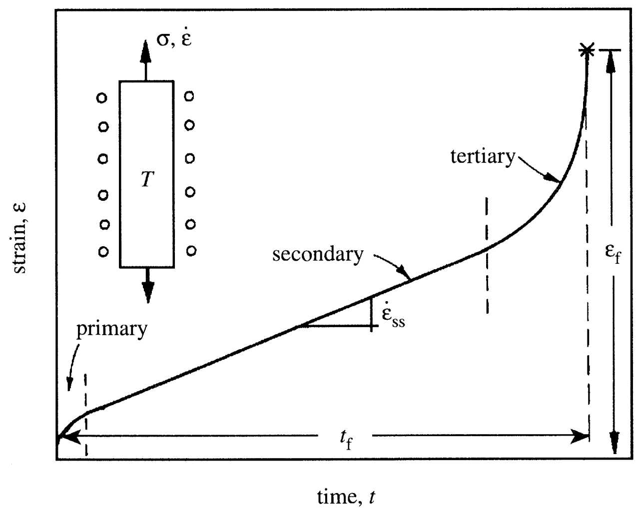

Figure 4. A typical creep curve showing the tensile strain ϵ plotted against time t. It shows the primary, secondary and tertiary stages, and the quantities $\dot{\epsilon}_{ss}$, ϵ_f and t_f.

The subset of materials with the greatest value of an index can be identified rapidly using such charts. Taking logarithms of (2.2), for instance, we find

$$\log(\sigma_y) = \tfrac{3}{2}\log(\rho) + \tfrac{3}{2}\log(M_2). \quad (2.7)$$

The equation defines a grid of lines of slope 3/2 on the chart, one for each value of M_2. The construction is illustrated in figure 3, from which it can be seen that certain fibre-reinforced composites and ceramics have the particularly large values of M_2 and so are the best choice for a light, strong beam. The index M_1 plots as a line of slope 1; M_3 plots as a line of slope 2; they identify the best materials for light strong ties and panels. Plotting these on figure 3 identifies the best subset for each application. The chart plus the indices allow optimal selection for each combination of function, constraint and objective.

The method is fully developed in texts and software (Ashby 1992; Cebon & Ashby 1992; CMS software 1994). Multiple selection stages are possible by using different slices through property space, and the shape of the cross section (important in bending, torsion and in resisting buckling) can be included in the indices. But this is all for room temperature. We return to our earlier question: can the method be extended to high temperatures?

3. Material indices for high temperature, creep-limited design

In design against creep, we seek the material and the shape which will carry the design loads without failure, for the design life at the design temperature. The meaning of 'failure' depends on the application. We distinguish four types of failure, illustrated in figure 1.

1. Displacement-limited applications, in which precise dimensions or small clearances must be maintained (as in the discs and blades of turbines), when design is based on creep rates $\dot{\epsilon}$ or displacement rates $\dot{\delta}$.

2. Rupture-limited applications, in which dimensional tolerance is relatively unimportant, but fracture must be avoided (as in pressure-piping), when design is based on time-to-failure t_f.

3. Stress-relaxation-limited applications in which an initial tension relaxes with time (as in the pretensioning of cables or bolts) when design is based on a characteristic relaxation time t_r.

4. Buckling-limited applications, in which slender columns or panels carry compressive loads (as in the upper wing skin of an aircraft, or an externally pressurized tube), when design is based on critical time-to-instability t_b.

To tackle any of these we need constitutive equations which relate the strain rate $\dot{\epsilon}$ or time-to-failure t_f for a material to the stress σ and temperature T to which it is exposed.

(a) *Constitutive equation for creep deflection*

When a material is loaded at a temperature above about one third of its absolute melting point, T_m, it creeps. Figure 4 shows the shape of the tensile creep curve at constant stress, σ, and temperature T. A primary extension is followed by a stage of steady-state creep, which ends in an accelerating tertiary stage. The important parameters are marked: the steady-state creep rate $\dot{\epsilon}_{ss}$, the time to fracture, t_f, and the creep ductility, ϵ_f.

The characteristics of the curve and the way it changes with temperature and stress are described mathematically by a constitutive equation (Finnie & Heller 1959; Hult 1966; Penny & Marriott 1971; Gittus 1975; Frost & Ashby 1972; Evans & Wilshire 1985). Many constitutive equations for creep-rate have been suggested, some purely empirical, some science-based; most are a mix of the two. Those most widely used in engineering design when deflection is important relate the steady-state strain rate $\dot{\epsilon}_{ss}$ to the tensile stress σ and the temperature T thus:

$$\dot{\epsilon}_{ss} = Af(\sigma)e^{-Q/RT}. \tag{3.1}$$

where A is a kinetic constant, Q an activation energy, R the gas constant and $f(\sigma)$ means 'a function of stress σ'. The function $f(\sigma)$ can be approximated, over restricted ranges of stress, by a power law (an approximation associated with the name of Norton), giving

$$\dot{\epsilon}_{ss} = A\left(\frac{\sigma}{\sigma_0}\right)^n e^{-Q/RT} = \dot{\epsilon}_0\left(\frac{\sigma}{\sigma_0}\right)^n. \tag{3.2}$$

where the constant A, the activation energy Q, the exponent σ_0 and the characteristic strength constant σ_0 are material properties. Considerable experience has accumulated in the use of Norton's law, which has the appeal that it allows analytical solutions to a wide range of engineering problems (see, for instance, Finnie & Heller 1959; Hult 1966; Penny & Marriott 1971). For this reason we shall use it even though, from a scientific point of view, it lacks a completely respectable pedigree.

(b) *Constitutive equation for creep fracture*

When fracture rather than deflection is design-limiting, creep is characterized instead by the time to fracture, t_f. It, too, can be described by a constitutive equation with features like those of (3.1). Here, again, a power law gives an adequate description over a restricted range of σ and T:

$$t_f = B\left(\frac{\sigma}{\sigma_f}\right)^q e^{Q_f/RT} = t_{f0}\left(\frac{\sigma}{\sigma_f}\right)^q. \tag{3.3}$$

with its own values of kinetic constants B, activation energy Q_f, exponent q and characteristic strength σ_f.

(c) *Constitutive equation for creep relaxation*

Creep relaxation requires a constitutive equation which combines creep and elastic response. For tension, and neglecting transient creep, it takes the form:

$$\dot{\sigma}/E = -\dot{\epsilon}_{ss} = -\dot{\epsilon}_0\left(\frac{\sigma}{\sigma_0}\right)^n. \tag{3.4}$$

where E is Young's modulus, $\dot{\epsilon}_{ss}$ is given by (3.2) and $\dot{\sigma}$ is the rate of change of stress with time. For the bending of a beam (as in figure 2) the equation becomes, instead

$$\dot{F}/S = \dot{\delta}_c, \tag{3.5}$$

where $\dot{F}$ is the rate of change of force F, S is the bending stiffness and $\dot{\delta}_c$ is the creep deflection rate of the beam. Similar expressions describe torsion and compression.

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4. Material selection in the creep regime

We wish to derive indices which parallel those of § 2, but for design when creep takes place. The immediate difficulty is that the 'strength' is now no longer a fixed material property, but depends on temperature and on the strain rate. The treatment below is kept as brief as possible. Fuller details can be found elsewhere (Abel & Ashby 1994).

(a) *Deflection-limited design*

Consider first the trivial case of a tensile member – a tie – of minimum weight, designed to carry a load F for a life t without deflecting more than δ at a temperature T. If the tie has length L, the steady strain rate must not exceed

$$\dot{\epsilon} = \delta / Lt. \tag{4.1}$$

Inserting this into the constitutive (3.2) for tensile creep and inverting gives

$$\sigma = F/A = \sigma_0 (\delta / \dot{\epsilon}_0 Lt)^{1/n}. \tag{4.2}$$

The objective is to minimize the mass of the tie. Solving for A and substituting this into (2.4) gives

$$m = L\rho F / \sigma_{\mathrm{D}}$$

with

$$\sigma_{\mathrm{D}} = \sigma_0 (\delta / \dot{\epsilon}_0 Lt)^{1/n}. \tag{4.3}$$

Thus the lightest tie whic meets the contraints of F, T, t and δ is that made of the material with the largest value of

$$M = \sigma_{\mathrm{D}} / \rho. \tag{4.4}$$

This is just (2.1) with σ_{y} replaced by σ_{D}, defined above; it contains both temperature and time.

The analysis of beams, shafts, pressure vessels (and such like) is a little more complex, but follows the same pattern. Consider, as an illustration, the cantilever beam of figure 2*b* carrying a load F, but now at a temperature such that it creeps. The objective, as before, is to make the beam as light as possible; the constraints (again as before) are that its length L and the proportions α of its cross section are fixed, and that it must support the load F for a time t at temperature T without deflecting more than δ. The design specification constrains the deflection rate, $\dot{\delta}$: it must not exceed δ/t. The deflection rate $\dot{delta}$ for a cantelever beam with end load F, creeping according to the constitutive (3.2) is

$$\dot{\delta} = \frac{2}{n+2} \dot{\epsilon}_0 L^2 \left\{ \frac{4FL}{\sigma_0} \left(\frac{2n+1}{2n} \right) \frac{1}{bh^{(2n+1)/n}} \right\}^n. \tag{4.5}$$

Eliminating b and h between the equations (2.4) and (4.5), using $b = \alpha h$, gives

$$m = \rho L^{(3n+3)/(3n+1)} \alpha^{(n+1)/(3n+1)} \left[\left(\frac{2n+1}{2n} \right) \frac{4FL}{\sigma_{\mathrm{D}}} \right]^{2n/(3n+1)}. \tag{4.6}$$

with σ_{D}, which we call the *design strength*, given by

$$\sigma_{\mathrm{D}} = \sigma_0 \left(\frac{n+2}{2} \frac{\dot{\delta}}{L\dot{\epsilon}_0} \right)^{1/n}. \tag{4.7}$$

Table 1. *Creep deflection*

(Time to deflect through δ ('life') $= t$; $\dot{\delta} \propto \delta/t$, $n \gg 1$. To minimize cost, replace ρ by $C_m\rho$ in the expressions for M; to minimize energy content, replace ρ by $q_m\rho$.)

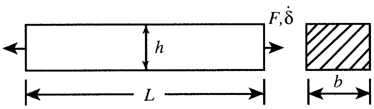
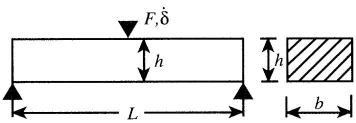
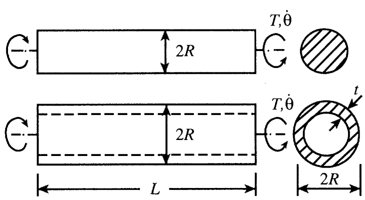
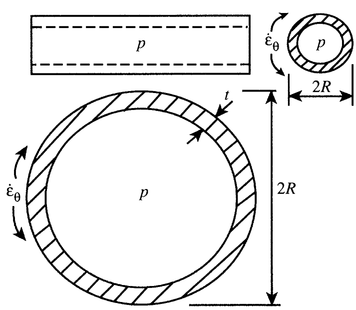
mode of loading	geometry	design strength	index
tension; centrifugal loading 	any b and h	$\sigma_D = \sigma_0 \left(\frac{\dot{\delta}}{L\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_D/\rho$
bending 	$b = \alpha h$	$\sigma_D = \sigma_0 \left(\frac{\dot{\delta}}{L\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_D^{2/3}/\rho$
	b fixed, h free		$M = \sigma_D^{1/2}/\rho$
	h fixed, b free		$M = \sigma_D/\rho$
torsion 	solid	$\sigma_D = \sigma_0 \left(\frac{\dot{\Theta}}{\sqrt{3}\,\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_D^{2/3}/\rho$
	R free, $t = \alpha R$		$M = \sigma_D^{2/3}/\rho$
	t fixed, R free		$M = \sigma_D^{1/2}/\rho$
	R fixed, t free	$\sigma_D = \sigma_0 \left(\frac{R\dot{\Theta}}{\sqrt{3}\,L\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_D/\rho$
pressure tubes and vessels 	R fixed, t free	$\sigma_D = \sigma_0 \left(\frac{\dot{R}}{R\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_D/\rho$

Table 2. *Creep fracture*

(Time to fracture ('life') = t_f; $n \gg 1$. To minimize cost, replace ρ by $C_m\rho$ in the expressions for M; to minimize energy content, replace ρ by $q_m\rho$.)

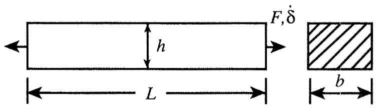
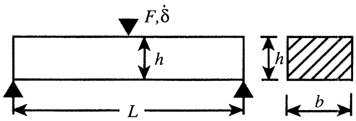
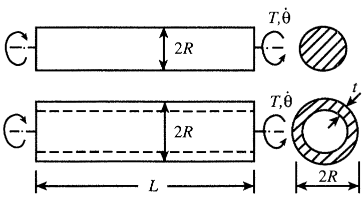
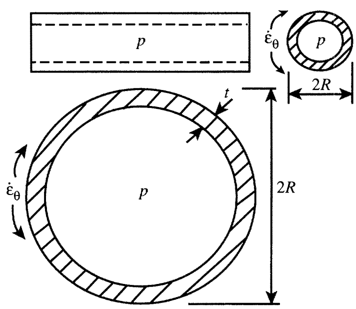
mode of loading	geometry	design strength	index
tension; centrifugal loading 	any b and h	$\sigma_\mathrm{D} = \sigma_\mathrm{f0}\left(\frac{t_\mathrm{f0}}{t_\mathrm{f}}\right)^{1/q}$	$M = \sigma_\mathrm{D}/\rho$
bending 	$b = \alpha h$	$\sigma_\mathrm{D} = \sigma_\mathrm{f0}\left(\frac{t_\mathrm{f0}}{t_\mathrm{f}}\right)^{1/q}$	$M = \sigma_\mathrm{D}^{2/3}/\rho$
	b fixed, h free		$M = \sigma_\mathrm{D}^{1/2}/\rho$
	h fixed, b free		$M = \sigma_\mathrm{D}/\rho$
torsion 	solid, R free	$\sigma_\mathrm{D} = \sigma_\mathrm{f0}\left(\frac{t_\mathrm{f0}}{t_\mathrm{f}}\right)^{1/q}$	$M = \sigma_\mathrm{D}^{2/3}/\rho$
	$t = \alpha R$		$M = \sigma_\mathrm{D}^{2/3}/\rho$
	t fixed, R free		$M = \sigma_\mathrm{D}^{1/2}/\rho$
	R fixed, t free		$M = \sigma_\mathrm{D}/\rho$
pressure tubes and vessels 	R fixed, t free	$\sigma_\mathrm{D} = \sigma_\mathrm{f0}\left(\frac{t_\mathrm{f0}}{t_\mathrm{f}}\right)^{1/q}$	$M = \sigma_\mathrm{D}/\rho$

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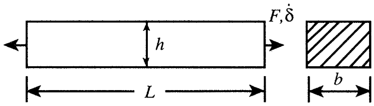
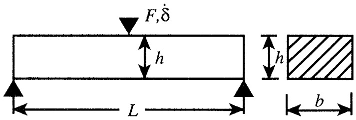
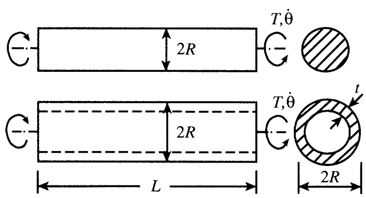
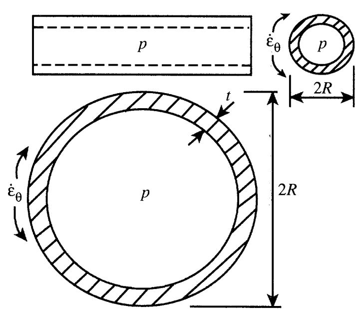
Table 3. *Creep relaxation*

(Time to relax to $\frac{1}{2}F_i$ or $\frac{1}{2}T_i$ ('life') $= t_\mathrm{r}$; $n \gg 1$. To minimize cost, replace ρ by $C_m\rho$ in the expressions for M; to minimize energy content, replace ρ by $q_m\rho$.)

mode of loading	geometry	design strength	index
tension; centrifugal loading 	any b and h	$\sigma_\mathrm{D} = \sigma_0 \left(\dfrac{\sigma_0}{E\dot{\epsilon}_0 t_\mathrm{r}}\right)^{1/n-1}$	$M = \sigma_\mathrm{D}/\rho$
bending 	$b = \alpha h$	$\sigma_\mathrm{D} = \sigma_0 \left(\dfrac{\sigma_0}{E\dot{\epsilon}_0 t_\mathrm{r}}\right)^{1/n-1}$	$M = \sigma_\mathrm{D}^{2/3}/\rho$
	b fixed h free		$M = \sigma_\mathrm{D}^{1/2}/\rho$
	h fixed b free		$M = \sigma_\mathrm{D}/\rho$
torsion 	solid R free $t = \alpha R$	$\sigma_\mathrm{D} = \sigma_0 \left(\dfrac{\sigma_0}{2\sqrt{3}\,\pi\dot{\epsilon}_0 G t_\mathrm{r}}\right)^{1/n-1}$	$M = \sigma_\mathrm{D}^{2/3}/\rho$
			$M = \sigma_\mathrm{D}^{2/3}/\rho$
	t fixed R free		$M = \sigma_\mathrm{D}^{1/2}/\rho$
	R fixed t free	$\sigma_\mathrm{D} = \sigma_0 \left(\dfrac{R\dot{\Theta}}{\sqrt{3}\,L\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_\mathrm{D}/\rho$
pressure tubes and vessels 	R fixed t free	$\sigma_\mathrm{D} = \sigma_0 \left(\dfrac{\dot{R}}{R\dot{\epsilon}_0}\right)^{1/n}$	$M = \sigma_\mathrm{D}/\rho$

Equation (4.6) looks messy, but it is not as bad as it seems. We examine first the fully plastic limit, found by setting $n = \infty$. The equation now simplifies to

$$m = \rho L \alpha^{1/3}(4FL/\sigma_D), \tag{4.8}$$

which is identical with (2.6) if σ_D is replaced by σ_y. The design strength in creep, then, plays the same role as the yield strength in room-temperature plasticity. In fact, (4.8) is a good approximation to the more complex result of (4.6) over the entire range of values of the exponent n $(3 < n < 20)$ normally encountered in metals and alloys. Inspection shows that the mass is minimized by maximizing the index M_2 given earlier (equation (2.2)) with replaced by (remember that contains temperature and deflection rate).

A parallel calculation for a panel (flat plate in bending) gives (2.3), again with σ_y replaced by σ_D, appropriately defined. Similar expressions are derived for torsion and for internal pressure (tables 1–3). Selection with the objective of minimizing cost rather than weight lead to identical results with ρ replaced by $C_m\rho$, where C_m is the material cost per kilogram; and the objective of minimizing energy content is achieved by replacing this by $q_m\rho$, where q_m is the energy content per kilogram.

(*b*) *Fracture-limited design*

Consider next an application in which fracture, not deflection rate, is design limiting (figure 1*b*). For a bending beam the largest stresses appear, and creep fracture starts, in the outer fibres at the place where the bending moment M is greatest (figure 2*c*). The time t to the onset of failure of the cantilever, using the constitutive relation of (3.3), is

$$t = t_{fo}\left(\frac{2q+1}{2q}\frac{4FL}{\sigma_{fo}bh^2}\right)^q. \tag{4.9}$$

Writing $b = \alpha h$, solving for the area $A = bh$ and substituting in (2.4) gives

$$m = \rho L \alpha^{1/3}\left[\left(\frac{2q+1}{2q}\right)\frac{4FL}{\sigma_D}\right]^{2/3}. \tag{4.10}$$

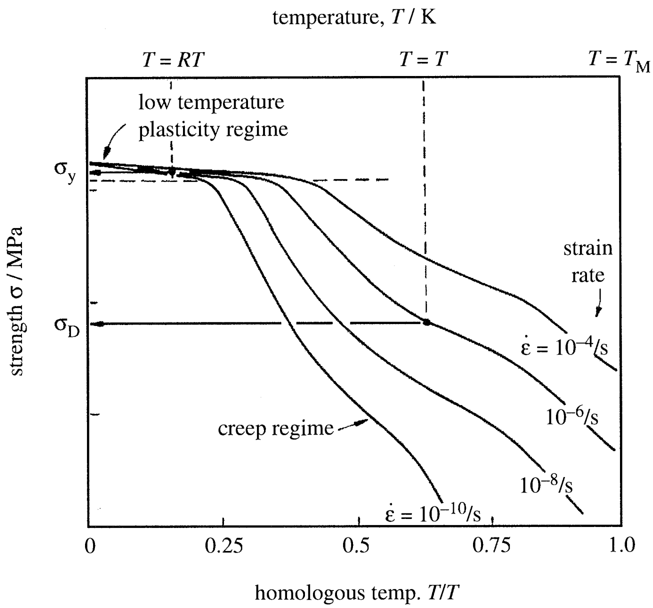
with the design strength

$$\sigma_D = \sigma_{fo}(t_{fo}/t)^{1/q}. \tag{4.11}$$

The parallel with deflection-limited design is obvious; and once again the result reduces to that for full plasticity in the limit $q = \infty$. The mass is minimized, as before, by maximizing the index of (2.2), with σ_y replaced by this new, fracture-related σ_D which depends on design life, t, and on temperature, T. Analagous calculations for ties and panels give (2.1) and (2.3) again; only the definition of σ_D is different. Similar expressions describe torsion and internal pressure (table 1), and are modified for cost or energy content by replacing ρ by $C_m\rho$ or $q_m\rho$, as before.

(*c*) *Relaxation-limited design*

A tensile cable or a bolt, pretensioned to provide a bearing or clamping force F at an elevated temperature, relaxes with time by creep. The calculation is a standard one; elastic strain σ/E is replaced over time by creep strain. The total change in strain in the cable or bolt is zero, since its ends are fixed. The governing



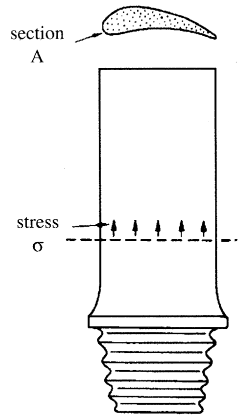


Figure 5. The strength, at room temperature, is measured by the yield strength σ_y, which has a small temperature dependence that is often ignored. At high temperatures, the strength σ_D depends strongly on temperature and strain rate.

Figure 6. A turbine blade.

equation for the stress in the component was given earlier as (3.4). Integrating this with the boundary condition $\sigma = \sigma_i$ at $t = 0$, gives

$$\left(\frac{\sigma_o}{\sigma}\right)^{n-1} - \left(\frac{\sigma_o}{\sigma_i}\right)^{n-1} = (n-1)\frac{E\dot{\epsilon}_0 t}{\sigma_o}, \tag{4.12}$$

where σ_i is the stress to which the cable or bolt was originally tightened, and σ is the stress to which it has relaxed in time t. In this case a constraint is specified by defining a characteristic relaxation time, t_r, as the time required for the stress to relax to a specified fraction of its initial value. Inverting (4.12) gives:

$$t_r = \left[1 - \left(\frac{\sigma}{\sigma_i}\right)^{n-1}\right]\frac{\sigma_0^n}{(n-1)E\dot{\epsilon}_0\sigma^{n-1}}. \tag{4.13}$$

We take this fraction σ/σ_i, to be 0.5, for the purpose of illustrating the method. The result is independent of the choice of value. For $n > 3$ the term in square brackets is then close to unity; for simplicity we shall neglect it. The design specifies the minimum bearing or clamping load F. Writing $\sigma = F/A$, and substituting for A in (2.4), gives the mass of the cable or bolt which will safely provide a clamping load greater than F for a life t_r:

$$m = L\rho F/\sigma_D \tag{4.14}$$

with

$$\sigma_D = \sigma_0\left(\frac{\sigma_0}{(n-1)E\dot{\epsilon}_0 t_r}\right)^{1/(n-1)}. \tag{4.15}$$

The mass is minimized by selecting the material with the greatest value of σ_D/ρ, that is, the material index is once more that of (2.1) with σ_y replaced by this new σ_D.

Springs, too, relax their tension with time. Most are loaded in bending, when the constitutive behaviour is that of (3.5). Taking a beam of length L as an example, we write, for the stiffness S:

$$S = C_2 EI/L^3, \tag{4.16}$$

where I is the second moment of its area and C_2 is a constant. Integrating with the boundary conditions $F = F_i$ at $t = 0$ gives a result with the form of (4.12). Proceeding as before, we find for the minimum weight design of a leaf spring (or any spring loaded in bending), which must not relax its restoring force in time t_r at temperature T, the index M_2 of (2.2), with σ_D (when $n > 3$) given by

$$\sigma_D \approx \sigma_0 \left(\frac{\sigma_0}{E\dot{\epsilon}_0 t_r} \right)^{1/(n-1)}. \tag{4.17}$$

Table 1 lists results for other modes of loading. Similar calculations for ties and panels give (2.1) and (2.3) again; only the definition of σ_D is different. The earlier adaptations to cost or energy apply here too.

5. The selection procedure

Expressions for the indices M and the associated design strengths σ_D are summarized in table 1. The close parallel between these results and those for room temperature plasticity (equations (2.1), (2.2) and (2.3)) suggests a selection procedure. The design temperature T and acceptable deflection rate $\dot{\delta}$, or life t, or relaxation time t_r are identified. Using this information, values for the appropriate σ_D are calculated from a database of creep properties for materials (it is necessary to cap the value of σ_D at the value σ_y to allow for the change of deformation mechanism to yielding at low temperatures). These are used to construct a chart of $\log(\sigma_D)$ against $\log(\rho)$; it is the creep equivalent of figure 3, but is specific to the particular temperature, deflection rate or life required by the design since these appear in the definition of σ_D. The indices M_1, M_2 and M_3 can be plotted onto it, allowing optimum selection for each application.

The easiest way to see how all this works is through examples. Those of the next section are deliberately simplified to avoid unnecessary digression. The method remains the same when the complexity is restored.

6. Specific applications

The examples below illustrate the selection of materials for structures loaded at elevated temperatures, and which are limited by deflection, by fracture or by stress relaxation. The selection is based on a single criterion: that of best dealing with one or other of these creep-related limits. Many other considerations enter the selection of materials for high-temperature use: resistance to oxidation, to thermal shock, and so on. Here we consider the selection for the initial shortlist, to which these other considerations can then be applied.

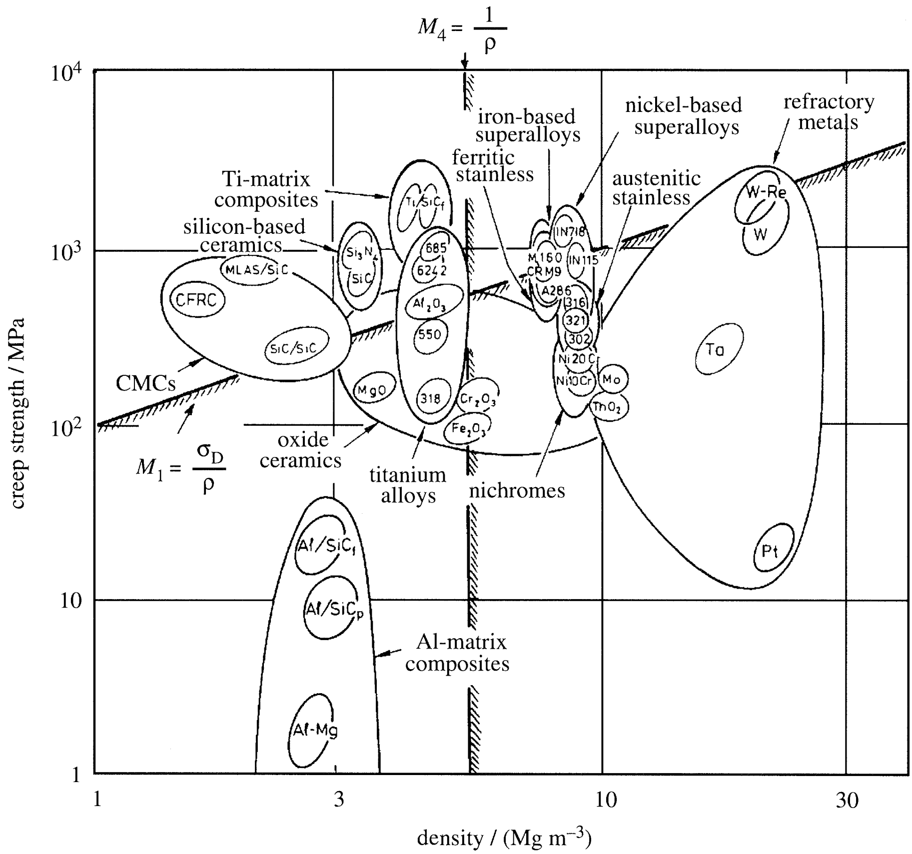


Figure 7. A selection chart for deflection-limited design at 400 °C for a life of 1000 h.

(a) *Fan and turbine blades for gas turbines*

A rotating blade of an aircraft turbine is self-loaded (figure 1*a* and 5); the centrifugal force caused by its own mass is much larger than that exerted by the gases which propel it. Adiabatic compression of the intake air can heat the compressor-fan blade to 400 °C or more. The dominant mode of steady loading, therefore, is tensile, and proportional (for fixed blade proportions) to the density of the material of which the blade is made. It could, then, be anticipated that the appropriate index is that for tensile loading, M_1 of (2.1), with the design strength for tensile loading, (4.3). More detailed analyses add complexity, but confirm this result (Able & Ashby 1994). The turbine blade is loaded in the same way, but is hotter: designers would like to go to 1000 °C. Concepts for new turbines push this temperature to 1500 °C. The task is to select materials to maximize the safe angular velocity of the fan or turbine blade, designed to operate for a life $t = 1000$ h without extending by more than δ, which is required to be 1.0% of its length, for each of these temperatures, and at the same time to minimize the weight.

The profile and section are determined by the blade design; neither is free. The mass is minimized by minimizing

$$M_4 = 1/\rho. \tag{6.1}$$

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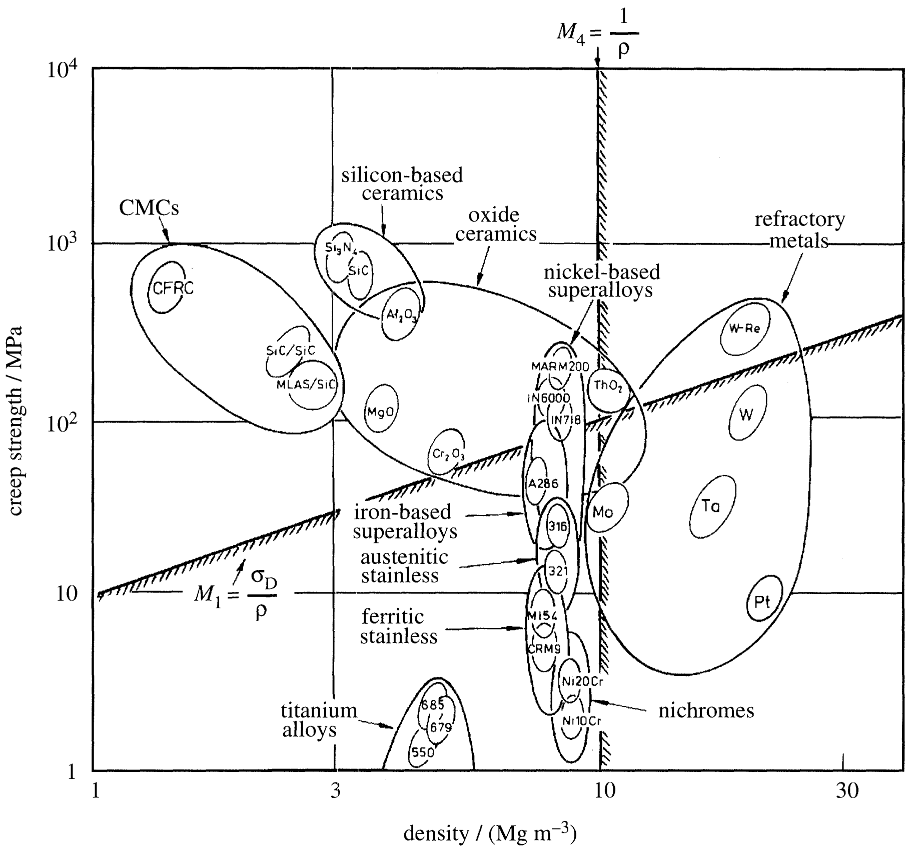

Figure 8. A selection chart for deflection-limited design at 1000 °C for a life of 1000 h.

Table 4. *Materials for fan blades at* 400 °C

material	comment
Ti-matrix composites (Ti/SiC$_f$)	the ultimate in performance, but very expensive
Ti alloys (e.g. Ti 685,6242)	high creep strength and low density makes these the best choice
Ni-based superalloys (e.g. IN738)	excellent creep strength, but both indices M_1 and M_4 inferior to Ti alloys
iron-based superalloys, stainless steels	less good than nickel-based alloys, but cheaper
ceramics and CMCs	excellent values of M_1 and M_4 but brittleness is a problem

Figures 6, 7 and 8 show σ_D, calculated for 400 °C, 1000 °C and and 1500 °C, with a value of δ/L and T corresponding to the design specification, plotted against density, ρ. Selection lines plotting the appropriate indices are shown. The selections are listed in tables 4, 5 and 6.

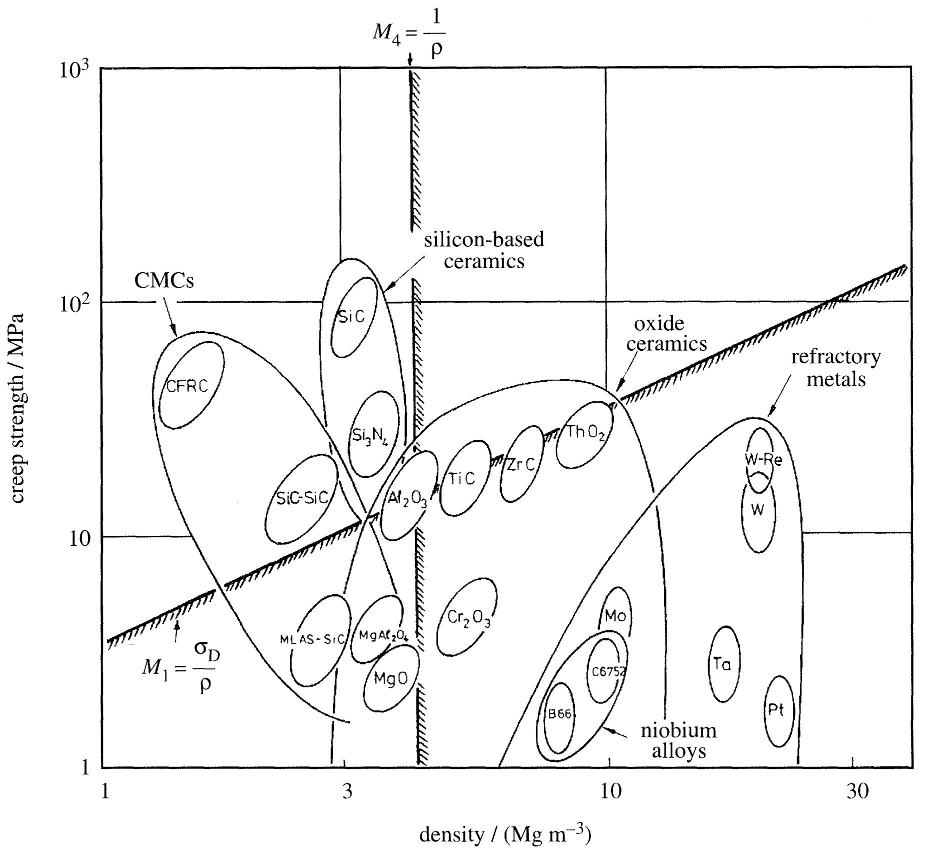

Figure 9. A selection chart for deflection-limited design at 1500 °C for a life of 1000 h.

Table 5. *Materials for fan blades at 1000* °C

material	comment
Ni-based superalloys	among metallic alloys, these have the highest values of M_1, but heavy
refractory metals	W–Re alloys offer high M_1 but very heavy
oxide ceramics and silicon-based ceramics, CMSs	large weight saving possible if design can accommodate brittleness

7. Summary and conclusions

A design-led procedure is proposed for the selection of materials to resist creep deflection, creep fracture, buckling and stress relaxation. It is an extension of a successful procedure for room temperature design which couples performance indices with materials-selection charts to identify an optimum subset of candidate materials for a given design. The extension requires the definition of a 'design strength', σ_D, which characterizes material response under conditions imposed by the design: the temperature, and the acceptable creep deflection, life or relaxation

Table 6. *Materials for fan blades at* 1500 °C

material	comment
oxide and silicon-based ceramics (SiC, Al_2O_3)	offer high values of M_1 and M_4 but design must accommodate brittleness
refractory metals	W–Re creep resistant, but heavy
ceramic composites (SiC–SiC; carbon–carbon)	potential for gains in performance, but brittleness and chemical stability require attention

time. Materials are ranked by an optimization procedure which combines σ_D with other properties (such as density, or cost, or stiffness) to isolate the subset which best meet the design specification.

The paper concludes with examples of the application of the method, which, when implemented in software, gives an efficient way of identifying promising candidates for a wide range of creep-limited application.

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References

Abel, C. A. & Ashby, M. F. 1994 Materials selection to resist creep and creep rupture. Report CUED/C-EDC/TR18, June, pp. 1–32. Engineering Department, Cambridge University.

Ashby, M. F. 1992 *Materials selection in mechanical design.* Oxford: Pergamon Press.

Cebon, D. & Ashby, M. F. 1992 Computer-aided materials selection for mechanical design. *Metals Mater.*, pp. 25–30.

CMS Software, Granta Design, 20 Trumpington Street, Cambridge CB2 1QA, UK.

Crane, F. A. A. & Charles, J. A. 1984 *Selection and use of engineering materials.* London: Butterworths.

Evans, R. W. & Wilshire, B. 1985 *Creep of metals and alloys.* London: The Institute of Metals.

Finnie, I. & Heller, W. R. 1959 *Creep of engineering materials.* New York: McGraw-Hill.

Frost, H. J. & Ashby, M. F. 1982 *Deformation mechanism maps.* Oxford: Pergamon Press.

Gittus, J. 1975 *Creep, viscoelasticity and creep fracture.* London: Elsevier.

Hult, J. 1966 *Creep in engineering structures.* London: Baisdell Press.

Penny, R. K. & Marriott, D. L. 1971 *Design for creep.* Boston, MA: McGraw-Hill.

Discussion

M. J. Goulette (*Rolls-Royce plc, Derby, UK*). The linkage of design systems and materials selector proposed by Professor Ashby may not be useful for final designs of gas turbines because the choice of materials for a specific component is very limited at this stage. It should prove to be useful, however, at earlier stages in the technology aquisition processes when choices have to be made about which competing material systems will be scaled up and fully characterized. At this stage the design linkage is vital so that choices which are correct both technically and commercially can be made.

M. F. Ashby. I agree; the procedure outlined here helps with the early stages of the design process.

A. Cottrell (*Department of Materials Science, University of Cambridge, UK*). It seems to me that Professor Ashby's excellent system for inter-relating material properties with engineering design, could be useful particularly for the designer who is aiming at revolutionary new engineering systems, as for example when the gas turbine was introduced in place of the piston engine, because with such radically new and unexplored engineering it will be important to know what materials might be most useful and there may be no previous experience in these cases to guide the design engineer. Professor Ashby's logical system of analysis would be a good way of making initial entries into such pioneering and unexplored fields.

G. A. Webster (*Department of Mechanical Engineering, Imperial College, London, UK*). I would like to relate Professor Ashby's procedure for selecting materials to resist creep deformation and failure to Professor McLean's remarks about there being significant scope for engineering exploitation in achieving optimum high-temperature designs. Professor Ashby illustrated the procedure using components of fixed dimensions. It may be possible to further refine materials choice by allowing the component dimensions to be altered. For example, turbine blades could be tapered and different disc profiles examined. Is it possible to incorporate these features into the selection procedure?

M. F. Ashby. Coupled selection of material and shape is possible: see Ashby (1992).

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